Writing Numbers in Exponential Notation

B9
This learning module consists of a review of *standard exponential notation*. After completing this module you should be able to:

- Write large and small numbers in exponential notation
- Convert numbers already in exponential form into standard exponential notation

Some simple and easy to remember rules will be presented to aid in the conversion of a number from one form to another.
Throughout this module, you will be given examples to work. Complete these examples on scratch paper. Make sure that you understand all the examples before proceeding. When you feel that you understand the material, obtain and complete a posttest from the Science Learning Center personnel. If you successfully complete the posttest, make sure your name is recorded in the SLC database. If you do not pass the posttest, you may review the module and retake the posttest as many times as needed.
Introduction

In science, as well as other disciplines, it is often necessary to work with extremely large or small numbers such as the following.

Speed of Light = 30,000,000,000 cm/sec

Number of Atoms in 1 Gram of Carbon = 50,150,000,000,000,000,000,000,000 atoms

Mass of One Hydrogen Atom = 0.000000000000000000000001674 gram

Solubility of Cadmium Sulfide Gas in Water = 0.00000000000000000883 mole/liter
A simple way to avoid the problems involved with writing and working with large and small numbers is to express them in *standard exponential notation* or *standard scientific notation* as shown below.

**Speed of Light:** $3.00 \times 10^{10}$ cm/sec

**Number of Atoms in 1 Gram of Carbon:** $5.015 \times 10^{22}$ atoms

**Mass of One Hydrogen Atom:** $1.674 \times 10^{-24}$ gram

**Solubility of Cadmium Sulfide Gas in Water:** $8.83 \times 10^{-14}$

From just observing the space occupied by these numbers now, you can see that it is more efficient to work with these numbers.
In order to make use of exponential notation, you must understand the *derivation* of both large and small numbers written in standard exponential notation. Consider the example below.

2173.0 is also equivalent to writing $2.1730 \times 1000$

or $2.1730 \times (10)(10)(10)$

and since $(10)(10)(10)=10^3$

we may also write $2.173 \times 10^3$

This large number is now in *standard exponential notation*!
Below is another example, but this time a small number is given. The same process has been carried out here as in the previous example except that instead of multiplying by a large number (1000 or $10^3$), we are now multiplying by a small number, 0.0001 or $10^{-4}$.

0.000716 is also equivalent to writing $7.16 \times 0.0001$

or $7.16 \times (0.1)(0.1)(0.1)(0.1)$

and since $0.1 = 10^{-1}$, and $(10^{-1})(10^{-1})(10^{-1})(10^{-1}) = 10^{-4}$

we may also write $7.16 \times 10^{-4}$

This small number is now in standard exponential notation!
The method of conversion to standard exponential notation is not the easiest method to use, but it does give you an idea of where an exponential comes from.

An easier method is outlined below.
From the previous two examples, a general rule can be seen. *In standard exponential notation, the number is always written with only one digit to the left of the decimal.*

Example:

The *correct* way to write 2,173 in standard exponential form is

\[2.1730 \times 10^3\]

The number 2,173 could also be written *incorrectly* as the following:

\[2,173 = 21.73 \times 10^2\]
\[2,173 = 217.3 \times 10^1\]
\[2,173 = 2173 \times 10^0\]
\[2,173 = 0.2173 \times 10^4\]
\[2,173 = 0.02173 \times 10^5\]

The above ways are *incorrect* because only *one digit* should be to the *left of the decimal.*
When we convert the number 2173.0 to standard exponential notation, we have to move the decimal 3 places to the left. As it turns out, this is the same number, 3, as the exponent on 10 when 2173.0 is in standard exponential form.

Another general rule can be derived from this example. *The exponent of 10 on a large number is equal to the number of places the decimal point has to be moved.*

**Example:**

2173.0

Decimal moves 3 places

2.1730 x 10^3

(Exponent on 10 is 3)
Another Example:

3,170,000,000.

Decimal moves 9 places

3.17 \times 10^9

(Exponent on 10 is 9)
When working with small numbers, the same general pattern develops. Below the example 0.000716 is worked out.

0.000716

Decimal moves 4 places

7.16 \times 10^{-4}

(Exponent on 10 is 4 with a negative sign)

As a general rule, the exponent of a small number is a negative number equal to the number of places the decimal point moves.
Rules for Writing Standard Exponential Notation

Another example is given below:

0.0000575

Decimal moves 5 places

5.75 \times 10^{-5}

(Exponent on 10 is 5, but with a negative sign)
The rules for writing a number in standard exponential notation will not change if the exponent is 0.

Example: Suppose you have a number, 5.134 and wish to write it in standard exponential notation. Your answer would be:

$$5.134 \times 10^0$$

Because $10^0 = 1$

So, $(5.134 \times 10^0) = (5.134 \times 1) = 5.134$
In summary, to convert a number to standard exponential notation:

1. Move the decimal point so that there is only one digit to the left of the decimal.

2. Count the number of places the decimal moves.

3. If the number is large (or the decimal moves to the left), the exponent on 10 is equal to the number of places the decimal was moved and has a positive sign.

4. If the number is small (or the decimal moves to the right), the exponent on 10 is equal to the number of places the decimal was moved and has a negative sign.

This process results in a number of the following general form:

\[ C \times 10^n \]

where \( C \) is the number between 1 and 9.999... and \( n \) is a positive or a negative integer.
Practice Problem Set 1:

Before proceeding, convert the following numbers to *standard exponential form* on a piece of scratch paper.

1) 3,760,000,000
2) 0.0000567
3) 0.000000091
4) 476,100

Check your answers. Make sure that you understand this process before proceeding.
Answers to Practice Problem Set 1:

1) $3.76 \times 10^9$
2) $5.67 \times 10^{-5}$
3) $9.1 \times 10^{-9}$
4) $4.761 \times 10^5$
Numbers that result from calculations are often not in standard exponential notation, and you will need to change the value of the exponent to have your resulting number in the correct standard form.
For example, you multiplied two numbers together and got the answer $81.6 \times 10^3$, which is *not* in standard exponential notation (with one digit to the left of the decimal). To convert this number to standard exponential notation, we need to move the decimal point to the left and increase the exponent on 10 by 1 as shown.

\[ 81.6 \times 10^3 \]

\[ 8.16 \times 10^{3+1} \]

(1 is added to the current exponent of 10)

\[ 8.16 \times 10^4 \]
Below is an example with a negative exponent of 10. We again add a number to the exponent of 10 equal to the number of places the decimal was moved to the left.

\[ 2040.0 \times 10^{-5} \]

Decimal moves 3 places to the left

\[ 2.040 \times 10^{(-5+3)} \]

(3 is added to the current exponent of 10)

\[ 2.040 \times 10^{-2} \]
If the number is small *and the decimal must move to the right*, we *subtract* the number of places the decimal moves from the *current exponent* as shown in the following examples.

\[ 0.0414 \times 10^{-3} \]

Decimal moves 2 places to the right

\[ 4.14 \times 10^{(-3-2)} \]

(2 is subtracted from the current exponent of 10)

\[ 4.14 \times 10^{-5} \]
In another example

\[ 0.00051 \times 10^9 \]

Decimal moves 4 places to the right

\[ 5.1 \times 10^{(9-4)} \]

(4 is subtracted form the current exponent of 10)

\[ 5.1 \times 10^5 \]
We can formulate two general rules:

1) If the decimal point moves to the left (or the number is large), add the number of places the decimal moves to the current exponent of 10.

2) If the decimal point moves to the right (or the number is small), subtract the number of places the decimal moves from the current exponent of 10.
Practice Problem Set 2:

Convert the following numbers to *standard exponential notation* on a piece of scratch paper.

1) $0.00416 \times 10^6$
2) $24.8 \times 10^{-3}$
3) $0.716 \times 10^{-4}$
4) $3410 \times 10^2$
Answers to Practice Problem Set 2:

1) $4.16 \times 10^3$
2) $2.48 \times 10^{-2}$
3) $7.16 \times 10^{-5}$
4) $3.41 \times 10^5$
Check Your Work for Reasonableness!

Finally, you should *always check calculations or conversions* of any kind to be certain they appear reasonable. You can check these conversions by *writing them out as regular numbers*.

In the previous examples, you converted $0.0052 \times 10^4$ to $5.2 \times 10^1$. Write out both of these numbers. They should be equal if the conversion was done correctly.

$$0.0052 \times 10^4 = 52$$

*Check!*

$$5.2 \times 10^1 = 52.$$
Another example:

\[ 714.24 \times 10^7 = 7,142,400,000 \]

Check!

\[ 7.1424 \times 10^9 = 7,142,400,000 \]
Final Directions

When you feel you understand the material presented in this module, obtain and complete a posttest. Have the test checked before you leave.

If you complete the test correctly, make sure your name is recorded in the Science Learning Center database. If you make any mistakes in the posttest you may review this module and retake the test as many times as necessary.

Good Luck!