Determining Significant Figures
A4
Science Learning Center
University of Michigan, Dearborn
Directions

This module consists of a written explanation and examples problems. As you read through the material, be sure to complete all example problems. When you have completed reading the module and you feel you fully understand the material, ask for a posttest. If you pass the posttest, make sure your name is recorded in the SLC database. If you do not pass it, you may review the module and retake the test as many times as needed.
Objectives

✓ Know what significant figures are.

✓ Know which numbers are significant.

✓ Be able to express the result of a measurement with the correct number of significant figures.

✓ Distinguish between significant and non-significant zeros.

✓ Be aware that the number of figures in scientific notation indicates precision.

✓ Be able to express the result of a calculation with the correct number of significant figures.
What are Significant Figures?

Significant Figures and Measurements

A significant figure is one that has *some significance* but *does not necessarily denote a certainty*.

Whenever you estimate any kind of measurement, for example the length or weight of an object, there is always a limit to the number of digits you can read.

The *number of significant figures* in a measurement is the *number of digits that are known with certainty plus the last one that is not absolutely certain*. 
What are Significant Figures?

Significant Figures and Measurements

As a general rule you should attempt to read any scale to one tenth of its smallest division by visual interpolation.

In the case below, you would read to ± 0.01cm. This estimated figure will always be your last significant figure with the implied accuracy of ± 1. Therefore, the measurement is written as 4.63 ± 0.01cm.

Generally, read any scale to 1/10 of smallest division.

![Image of a ruler showing measurement 4.63 ± 0.01 cm]
A length measurement of 5.63 cm contains *three significant figures*. The first two, the 5 and 6, are *certain*. The last digit, the 3, is *uncertain*. The uncertainty in the last significant figure is usually $\pm 1$. The result is $5.63 \pm 0.01\text{cm}$.

An Analytical Balance is *precise to four decimal places* with an *uncertainty of $\pm 1$* in the last significant figure. Therefore, the measurement $13.7654\text{g}$ is written as $13.7654 \pm 0.0001\text{g}$ and has six significant figures.
Here we see another kind of measurement, the reading of the position of a buret meniscus, (the curved liquid surface in a buret). The liquid level is always read at the bottom of the meniscus for transparent liquids.

**The reading in this buret is 12.75.** Four significant figures are implied. The last significant figure, 5, is obtained by visual interpolation between the 0.1 milliliter divisions.

All observers should agree with the first three significant figures but not necessarily with the last figure recorded here. *Disagreements of ±1 in the last digit are expected with visual interpolations.* The measurement is, therefore, written as 12.75 ± 0.01ml and has four significant figures.
What are Significant Figures?

Significant Figures and Measurements

When reading a measurement from a meter, you should also read to one digit past the smallest division on the meter.

On this meter, the reading should be 1.27. The 1 and the 2 are certain. The 7 must be estimated visually. The measurement is written as $1.27 \pm 0.01\text{g}$.

The 7 is estimated and, therefore, uncertain.
Writing Significant Figures

During any calculation—addition, subtraction, multiplication, or division—your answer could be expressed *with too few or too many significant figures*.

These numeric values may imply a *precision* that does not exist in the experiment being evaluated.

If you round off incorrectly, your answer will have an incorrect number of significant figures and will lose precision.

<table>
<thead>
<tr>
<th>EXAMPLE</th>
<th>1.024 x 1.2 = 1.2288</th>
<th>1.024 x 1.2 = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Too many numerals</td>
<td></td>
<td>Too few numerals</td>
</tr>
<tr>
<td>Too precise</td>
<td></td>
<td>Not precise enough</td>
</tr>
</tbody>
</table>

We, therefore, have developed rules for determining the correct number of significant figures in a number and apply these rules to calculations.
Writing Significant Figures

First we need to learn how to evaluate the number of significant figures a given number contains. This is necessary for calculations such as addition, subtraction, multiplication and division.

✓ Any written number that is not a zero is significant.

In this table the significant figures are underlined:

<table>
<thead>
<tr>
<th>3 significant figures</th>
<th>4 significant figures</th>
<th>5 significant figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.4</td>
<td>345.6</td>
<td>678.90</td>
</tr>
<tr>
<td>2.34</td>
<td>3.456</td>
<td></td>
</tr>
<tr>
<td>0.234</td>
<td>0.03456</td>
<td></td>
</tr>
</tbody>
</table>

Note: The zeros in 0.234 and 0.03456 are not significant, but the zero in 678.90 is a significant figure. Zeros have special rules as we will discuss in the next several slides.
Writing Significant Figures

Standard Exponential Notation and Zeros

✓ Zeros appearing between nonzero numbers are significant.

<table>
<thead>
<tr>
<th>Examples</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>40.7 L</td>
<td>3 significant figures</td>
</tr>
<tr>
<td>87,009 km</td>
<td>5 significant figures</td>
</tr>
</tbody>
</table>
Writing Significant Figures

Standard Exponential Notation and Zeros

✓ Zeros appearing in front of nonzero numbers are not significant.

<table>
<thead>
<tr>
<th>Examples</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.095987 m</td>
<td>0.0009 kg</td>
</tr>
<tr>
<td>5 significant figures</td>
<td>1 significant figure</td>
</tr>
</tbody>
</table>
Writing Significant Figures
Standard Exponential Notation and Zeros

✓ Zeros appearing at the end of a number and to the right of a decimal point are significant.

<table>
<thead>
<tr>
<th>Examples</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>850.00 g</td>
<td>9.500000000 mm</td>
</tr>
<tr>
<td>5 significant figures</td>
<td>10 significant figures</td>
</tr>
</tbody>
</table>
Writing Significant Figures
Standard Exponential Notation and Zeros

Without a decimal, large numbers containing zeros, such as 45,600 grams, pose a special problem. As the number is written, *we cannot tell whether the two zeros indicate the precision of the measurement or whether the zeros merely locate the decimal point.*

- If the zeros indicate precision, they are significant and the implied uncertainty is ± 1. This means that the measurement lies between 45,599 and 45,601.

- If, however, the zeros merely locate the decimal point and are not significant, the implied uncertainty is ± 100. Then we know the measurement lies between 45,500 and 45,700.

Thus a number written in this form is ambiguous.

<table>
<thead>
<tr>
<th>Example</th>
<th>Zeros Significant</th>
<th>Zeros Not Significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>45,600 grams</td>
<td>45,600 grams + 1 gram</td>
<td>45,600 grams + 100 grams</td>
</tr>
<tr>
<td></td>
<td>45,599</td>
<td>45,500</td>
</tr>
<tr>
<td></td>
<td>45,601</td>
<td>45,700</td>
</tr>
</tbody>
</table>

5 significant figures

3 significant figures

**For the purposes of this module and its posttest, assume the zeros are not significant for ambiguous numbers.**
Zeros appearing after a nonzero number which are not followed by another significant figure or a decimal point are **not** significant.

<table>
<thead>
<tr>
<th>Examples</th>
<th>85,000</th>
<th>85,000,000</th>
<th>85,000,000.0</th>
<th>85,000,000.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 significant figures</td>
<td>2 significant figures</td>
<td>9 significant figures</td>
<td>8 significant figures</td>
</tr>
</tbody>
</table>

**Note:** The decimal point after the zeros indicates that all the numbers are significant.
Writing Significant Figures

Summary: Rules for Zeros

✓ Zeros appearing between two nonzero numbers are significant.

Examples: The number 2.035 has 4 significant figures
The number 3,0007 has 5 significant figures.

✓ Zeros appearing in front of nonzero numbers are not significant.

Example: The number 0.0567 has only 3 significant figures.

✓ Zeros appearing at the end of a number and to the right of a decimal point indicate precision and are significant.

Example: 4.700 has 4 significant figures.

✓ Zeros appearing after a nonzero number which are not followed by another significant figure or a decimal point are not significant.

Example: The number 1200 has 2 significant figures, the number 1200. has 4 significant figures, and the number 1200.0 has 5 significant figures.
Writing Significant Figures

Standard Exponential Notation and Zeros

Ambiguity about the precision and number of significant figures for a particular number may be avoided by expressing the number in standard exponential notation or scientific notation.

- If the number 45,600.0 contains 5 significant figures, the number would be expressed as $4.5600 \times 10^4$. This notation implies 5 significant figures.

- If only 3 numbers are significant, the number would be expressed as $4.56 \times 10^4$.

<table>
<thead>
<tr>
<th>Normal notation</th>
<th>Scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>45,600</td>
<td>$4.5600 \times 10^4$</td>
</tr>
<tr>
<td>4,5600</td>
<td>$4.56 \times 10^4$</td>
</tr>
</tbody>
</table>
Scientific notation is also useful for clearly expressing very large and very small numbers with the correct precision and number of significant figures.

- The number 0.001230 can be expressed as $1.230 \times 10^{-3}$ which contains 4 significant figures.

- The number 900.00 can be expressed as $9.0000 \times 10^2$ which contains 5 significant figures.

- Finally, the number 2000. has a decimal on the end indicating precision and that all of the zeros are significant. Thus, 2000. can be expressed as $2.000 \times 10^3$ which contains 4 significant figures.

<table>
<thead>
<tr>
<th>Normal notation</th>
<th>Scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001230</td>
<td>$1.230 \times 10^{-3}$</td>
</tr>
<tr>
<td>900.00</td>
<td>$9.0000 \times 10^2$</td>
</tr>
<tr>
<td>2000.</td>
<td>$2.000 \times 10^3$</td>
</tr>
</tbody>
</table>
Practice Problems 1

How many significant figures are in each of the following numbers?

a) 1.234
b) 1.2340
c) $1.234 \times 10^{-3}$
d) $1.2340 \times 10^{-3}$
e) 1234
f) 12340
g) 0.012340
h) 10234
Solutions to Practice Problems 1

a) \( 1.234: 4 \)
b) \( 1.2340: 5 \)
c) \( 1.234 \times 10^{-3}: 4 \)
d) \( 1.2340 \times 10^{-3}: 5 \)
e) \( 1234: 4 \)
f) \( 12340: 4 \)
g) \( 0.012340: 5 \)
h) \( 10234: 5 \)
Calculations

You have learned how to determine how many significant figures are in a number. Now you will learn how many significant figures should be expressed in the result of a calculation.
Calculations
Adding and Subtracting

When *adding or subtracting* quantities, the rule is to *determine which number in the calculation has the least number of digits to the right of the decimal point*. Your result will have that same number of digits to the right of the decimal point.
### Calculations

#### Adding and Subtracting

<table>
<thead>
<tr>
<th>Example</th>
<th>26.46</th>
<th>Least number of digits to the right of the decimal = 2</th>
<th>26.46</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+ 4.123</td>
<td></td>
<td>- 4.123</td>
</tr>
<tr>
<td></td>
<td>30.583</td>
<td></td>
<td>22.337</td>
</tr>
<tr>
<td></td>
<td>30.58</td>
<td>Round to 2 decimals</td>
<td>22.34</td>
</tr>
</tbody>
</table>

For example, when adding 26.46 to 4.123, the calculated sum is **30.583**.

The original number 26.46 has the least number of digits to the right of the decimal point, two, so **the calculated sum is rounded to 30.58**.

In subtracting 4.123 from 26.46, the calculated difference is **22.337**.

The original number 26.46 has the least number of digits to the right of the decimal point, two, so **the calculated difference is rounded to 22.34**.
Calculations
Adding and Subtracting

<table>
<thead>
<tr>
<th>Example</th>
<th>2.634</th>
<th>+0.02</th>
<th>Least digits to the right = 2</th>
<th>-0.02</th>
<th>2.654</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.634</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.654</td>
</tr>
<tr>
<td>+0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.654</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Another example of addition is 2.634 plus 0.02. The calculated sum, 2.654, will be rounded off to 2.65.

**The number 0.02 has the least number of digits to the right of the decimal point, two, regardless of the fact that only one of the digits is significant.**

The result will have two digits to the right of the decimal point.

When subtracting 2.634 minus 0.02, the calculated difference, 2.614, is rounded off to 2.61.
Practice Problems 2

Complete the following arithmetic operations and express the answer with the correct number of significant figures:

a) \(1.421 + 0.4372 = \)
b) \(0.0241 + 0.11 = \)
c) \(0.14 + 1.2243 = \)
d) \(760.0 + 0.011 = \)
e) \(1.0123 - 0.002 = \)
f) \(123.69 - 20.1 = \)
g) \(463.231 - 14.0 = \)
h) \(47.2 - 0.01 = \)
Solutions to Practice Problems 2

a) $1.421 + 0.4372 = 1.858$

b) $0.0241 + 0.11 = 0.13$

c) $0.14 + 1.2243 = 1.36$

d) $760.0 + 0.011 = 760.0$

e) $1.0123 - 0.002 = 1.010$

f) $123.69 - 20.1 = 103.6$

g) $463.231 - 14.0 = 449.2$

h) $47.2 - 0.01 = 47.2$
Calculations

Multiplying and Dividing

For the multiplication and division of numbers, there is a different rule for determining the number of significant figures. When multiplying or dividing, determine which number entering the calculation has the smallest total number of significant figures regardless of the position of the decimal point. Your calculated result will have that same number of significant figures.
Calculations
Multiplying and Dividing

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least number of significant figures</td>
</tr>
<tr>
<td>2.61 x 1.2 = 3.132</td>
</tr>
<tr>
<td><strong>Round to: 3.1</strong></td>
</tr>
</tbody>
</table>

✓ In the example above, 2.61 multiplied by 1.2, the result is rounded off to **3.1**.

The number 1.2 has the *least number of significant figures*, two. So the calculated answer will also have two significant figures.

✓ When 2.61 is divided by 1.2, the result is also rounded off to two *significant figures*.

The calculated answer, 2.175, is rounded off to **2.2**.
Practice Problems 3

Perform the indicated operations. Express your answers with the **correct number of significant figures**:

a) \( 42.3 \times 2.61 = \)
b) \( 0.61 \times 42.1 = \)
c) \( 46.1 / 1.21 = \)
d) \( 23.2 / 4.1 = \)
Solutions to Practice Problems 3

a) $42.3 \times 2.61 = 110.$
b) $0.61 \times 42.1 = 26$
c) $46.1 / 1.21 = 38.1$
d) $23.2 / 4.1 = 5.7$
## Review of Rules for Calculations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Rule</th>
<th>Example</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition/Subtraction</td>
<td>Determine which number in the calculations has the least number of digits to the right of the decimal point. Your result will also have the same number of digits to the right of the decimal point.</td>
<td>234.7 + 1.623</td>
<td>236.323</td>
<td>Result: 236.3</td>
</tr>
<tr>
<td>Multiplication/Division</td>
<td>Determine which number in the calculations has the least total number of significant figures (regardless of the decimal point's position). Your result will also have that same number of significant figures.</td>
<td>44.2 x 2.662</td>
<td>117.6604</td>
<td>Result: 117</td>
</tr>
</tbody>
</table>
Additional Principles

As you begin to apply the principles of significant figures to actual problems or laboratory experiments, three additional principles should be presented.

**Principle 1:**
If you are using *exact constants*, *they do not affect the number of significant figures in your answer.*

For example, you might need to calculate how many feet equal 26.1 yards. The conversion factor you would need to use, 3 ft/yard, is an *exact constant* and *does not affect the number of significant figures* in your answer.

26.1 yards multiplied by 3 feet per yard equals 78.3 feet which has *3 significant figures.*

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXACT CONSTANT: 3 ft = 1 yd</td>
</tr>
<tr>
<td>26.1 yd</td>
</tr>
<tr>
<td>3 significant figures</td>
</tr>
</tbody>
</table>
Principle 2:
If you are using constants which are not exact (such as $\pi = 3.14$ or $3.142$ or $3.14159$) select the constant that has at least one or more significant figures than the smallest number of significant figures in your original data. This way the number of significant figures in the constant will not affect the number of significant figures in your answer.

For example, if you multiply 4.136 ft., which has four significant figures, times $\pi$, you should use 3.1416 which has 5 significant figures for $\pi$ and your answer will have 4 significant figures.

<table>
<thead>
<tr>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.316$ ft. $\times \pi = 4.316$ ft. $\times 3.1415 = 13.56$</td>
</tr>
</tbody>
</table>

4 significant figures

5 significant figures for $\pi$

answer in 4 significant figures
Additional Principles

Principle 3:
When you are doing several calculations, carry out all of the calculations to at least one more significant figure than you need. When you get the final result, then round off.

For example, you would like to know how many meters per second equals 55 miles per hour. The conversion factors you would use are: 1 mile = $1.61 \times 10^3$ meters and 1 hour = 3600 seconds. Your answer should have two significant figures. Your result would be $88.55$ divided by $3600$ which equals $24.59$ m/sec. This rounds off to $25$ m/sec.

By carrying this calculation out to at least one extra significant figure, we were able to round off and give the correct answer of $25$ m/sec rather than $24$ m/sec.

**EXAMPLE**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>How many meters per second is 55 miles per hour?</td>
<td></td>
</tr>
<tr>
<td>1 mile = $1.61 \times 10^3$ m (not exact constant; 3 sig. figs.)</td>
<td></td>
</tr>
<tr>
<td>1 hour = 3600 seconds (exact constant; 4 sig. figs.)</td>
<td></td>
</tr>
<tr>
<td>55 miles / hour = 55 miles / 1 hr. x $1.61 \times 10^3$ m / 1 mile x 1 hour / 3600 sec</td>
<td>$24.597$ m / 1 sec $= 25$ m/s</td>
</tr>
</tbody>
</table>
Take the Posttest!

You are now *finished* with this module. If you haven't already done the practice problems, we recommend you try them.

When you're done, obtain a *posttest* from the Science Learning Center personnel and complete it.

**GOOD LUCK!**