Calculation of Exponential Numbers

Practical Mathematics for Science Students, Program 802
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Calculation of Exponential Numbers is a written learning module which includes practice problems and a posttest (obtained in the Science Learning Center).
The objectives are to learn to:

1) convert numbers to standard exponential numbers,

2) change the exponent of a number to a higher or lower value without changing the magnitude of the exponential number, and

3) add, subtract, multiply and divide exponential numbers.
Before attempting this program, be sure that you thoroughly understand the conversion of decimal numbers to standard exponential notation.

First we will briefly review the conversion of numbers to exponential notation.
On the next slide, the number 1,234 is shown in six forms. The value of each form is the same.

As the decimal point is moved to the left, the value of the exponent increases by one for each move.

The standard exponential form of this number is $1.234 \times 10^3$. In this form the pre-exponential number lies between one and ten.

To add and subtract exponents, it is frequently necessary to change the form of one or more numbers.
1,234 =

12340.0 \times 10^{-1}

1234.0 \times 10^{0}

123.4 \times 10^{1}

12.34 \times 10^{2}

1.234 \times 10^{3} \quad \checkmark

0.1234 \times 10^{4}
1.234 \times 10^{-3} \text{ has a negative exponent.}

It can be changed to \( 0.1234 \times 10^{-2} \) which is a larger exponent, by using the same process. \underline{Remember} \( 10^{-2} \) is greater than \( 10^{-3} \).

This is shown on the next slide.
\[ 1.234 \times 10^4 = 1.234 \times 10^4 + 1 = 0.1234 \times 10^5 \]

\[ 1.234 \times 10^{-3} = 1.234 \times 10^{-3} + 1 = 0.1234 \times 10^{-2} \]

Remember \(10^{-2}\) is greater than \((>)\) \(10^{-3}\)
1.234 \times 10^5 = 1.234 \times 10^5 - 1 = 12.34 \times 10^4

1.234 \times 10^{-3} = 1.234 \times 10^{-3} - 1 = 12.34 \times 10^{-4}
Now, it is your turn to attempt some problems like these. Complete the problems below. The answers are on the next slide.

**Problem Set 1**

**Part A**
Insert the correct value of the exponent in place of the (?) question mark.

\[ = 8,765.4 \times 10^? \] (a)
\[ 87,654 = 876.54 \times 10^? \] (b)
\[ = 8.7654 \times 10^? \] (c)

Identify which of the above answers, a, b, or c, is in standard exponential form.

**Part B**
What is the correct value of the pre-exponential number?

\[ 7.777 \times 10^{-4} = ? \times 10^{-3} \]
\[ = ? \times 10^{-1} \]

**Part C**
Insert the correct value of the exponent in place of the question mark (?).

\[ 6,666.0 = 6.6660 \times 10^? \]
\[ = 0.066660 \times 10^? \]

**Part D**
Insert the correct value of the exponent in place of the question mark (?).

\[ 0.6666 = 666.60 \times 10^? \]
\[ = 0.0066660 \times 10^? \]
Solutions to Problem Set 1

Part A
Insert the correct value of the exponent in place of the (?) question mark.

\[ = 8,765.4 \times 10^1 \text{ (a)} \]

\[ 87,654 \quad = 876.54 \times 10^2 \text{ (b)} \]

\[ = 8.7654 \times 10^4 \text{ (c)} \]

Identify which of the above answers, a, b, or c, is in standard exponential form.

**Answer c** \((8.7654 \times 10^4)\) is in standard exponential form.

Part B
What is the correct value of the pre-exponential number?

\[ 7.777 \times 10^{-4} \quad = 0.7777 \times 10^{-3} \]

\[ = 0.007777 \times 10^{-1} \]

Part C
Insert the correct value of the exponent in place of the question mark (?).

\[ 6,666.0 \quad = 6.6660 \times 10^3 \]

\[ = 0.066660 \times 10^5 \]

Part D
Insert the correct value of the exponent in place of the question mark (?).

\[ 0.6666 \quad = 666.60 \times 10^{-3} \]

\[ = 0.0066660 \times 10^2 \]
Addition and Subtraction of Exponential Numbers

To add two exponential numbers you must first convert the exponents to the same value. It is best to change the exponent of the smaller number to that of the larger number.

In this example we will add $1.2 \times 10^4$ to $3.4 \times 10^5$. Using the rule, we first change the lower exponent to $10^5$. Both numbers will now have the same exponent: $0.12 \times 10^5$ and $3.4 \times 10^5$. We now add the pre-exponential numbers $0.12$ and $3.4$. Our answer is $3.52 \times 10^5$ which is rounded to $3.5 \times 10^5$. 

ADDING TWO EXPONENTIAL NUMBERS:

- convert exponents to same value
- best to change exponent of the smaller number to that of the larger number.

**EXAMPLE:**

\[(1.2 \times 10^4) + (3.4 \times 10^5)\]

\[
\begin{align*}
1.2 \times 10^{4+1=5} \\
(0.12 \times 10^5) + (3.4 + 10^5) \\
(0.12 + 3.4) \times 10^5 \\
= 3.52 \times 10^5 \text{ round to } 3.5 \times 10^5 \\
(2 \text{ significant figures})
\end{align*}
\]
SUBTRACTING TWO EXPONENTIAL NUMBERS:

- Again convert to the same exponent by changing the value of the lower exponent to that of the higher exponent.

**EXAMPLE:**

\[(3.4 \times 10^5) - (1.2 \times 10^4) =\]

ADD ONE

\[1.2 \times 10^{4+1=5}\]

MOVE ONE

\[(3.4 - 0.12) \times 10^5\]

= 3.28 \times 10^5 \text{ round to } 3.3 \times 10^5 \text{ (2 significant figures)}
ADDITON AND SUBTRACTION OF EXPONENTIAL NUMBERS WITH NEGATIVE EXPONENTS

REMEMBER - THE SMALLER THE EXPONENT, THE LARGER THE NUMBER.

THUS:

\[(1.2 \times 10^{-6}) + (3.2 \times 10^{-7}) = \]

ADD ONE

\[\times 3.2 \times 10^{-7+1=-6}\]

MOVE ONE

= \((1.2 + 0.32) \times 10^{-6}\)

= \(1.52 \times 10^{-6}\) round to \(1.5 \times 10^{-6}\)

(2 significant figures)

TO SUBTRACT:

\[(5.6 \times 10^{-6}) - (3.4 \times 10^{-7}) = \]

ADD ONE

\[\times 3.4 \times 10^{-7+1=-6}\]

MOVE ONE

= \((5.6 - 0.34) \times 10^{-6}\)

= \(5.26 \times 10^{-6}\) round to \(5.3 \times 10^{-6}\)

(2 significant figures)
Now, it is your turn to attempt some problems like these. Complete the problems below. The answers are on the next slide.

**Problem Set 2**

Add or subtract the following exponential numbers. Express all your answers in standard exponential form.

(a) $4.773 \times 10^4 + 5.62 \times 10^5 = ?$
(b) $2.612 \times 10^4 - 1.11 \times 10^3 = ?$
(c) $1.72 \times 10^{-4} + 2.14 \times 10^{-5} = ?$
(d) $8.12 \times 10^8 + 1.20 \times 10^6 = ?$
(e) $6.21 \times 10^{-2} - 1.7 \times 10^{-4} = ?$
(f) $1.2411 \times 10^2 - 1.2 \times 10^{-1} = ?$
(g) $3.14 \times 10^{-9} - 2.0 \times 10^{-11} = ?$
(h) $10^5 - 10^4 = ?$

**HINT:** Write $10^5$ as $1.0 \times 10^5$, and $10^4$ as $1.0 \times 10^4$)
Solutions to Problem Set 2

Add or subtract the following exponential numbers. Express all your answers in standard exponential form.

(a) $4.773 \times 10^4 + 5.62 \times 10^5 = 6.0973 \times 10^5$ round to $6.10 \times 10^5$

(b) $2.612 \times 10^4 - 1.11 \times 10^3 = 2.501 \times 10^4$

(c) $1.72 \times 10^{-4} + 2.14 \times 10^{-5} = 1.934 \times 10^{-4}$ round to $1.93 \times 10^{-4}$

(d) $8.12 \times 10^8 + 1.20 \times 10^6 = 8.132 \times 10^8$ round to $8.13 \times 10^8$

(e) $6.21 \times 10^{-2} - 1.7 \times 10^{-4} = 6.193 \times 10^{-2}$ round to $6.19 \times 10^{-2}$

(f) $1.2411 \times 10^2 - 1.2 \times 10^{-1} = 1.2399 \times 10^2$

(g) $3.14 \times 10^{-9} - 2.0 \times 10^{-11} = 3.12 \times 10^{-9}$

(h) $10^5 - 10^4 = 9.0 \times 10^4$
Multiplication and Division of Exponential Numbers

Now let’s consider the multiplication of exponentials. First we will consider an example using simple exponentials such as 10^2 times 10^3.

10^2 is 10x10 and 10^3 is 10x10x10; multiply together to get 10 x 10 x 10 x 10 x 10 x 10.

Thus 10^2 times 10^3 is 10^5 which is the sum of the two exponents 2 plus 3 or 10^{(2+3)}. 
EXAMPLE

10² x 10³ = ?

= 10² x 10³

= (10 x 10) x (10 x 10 x 10)

= 10 x 10 x 10 x 10 x 10

= 10⁵ (10² + 3)
MULTIPLYING EXPONENTS ...

ADD EXPONENTS

\[ 10^2 \times 10^3 = 10^{(2 + 3)} = 10^5 \]

\[ 10^4 \times 10^7 \times 10^2 = 10^{(4 + 7 + 2)} = 10^{13} \]
EXAMPLE: $10^5/10^2 = 10^3$ (DIFFERENCE 5-2=3)

DIVIDE = $\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10} = 10^3$
SIMPLIFYING THE DIVISION OF A NEGATIVE EXPONENTIAL

The division of exponentials can be simplified by recalling that a negative exponent can be written as the reciprocal of that number. $1/10^2$ is the same as $10^{-2}$. Thus $10^5$ divided by $10^2$ equals $10^5 \times 1/10^2$ or $10^5 \times 10^{-2}$. To multiply, we add exponents. $10^{(5-2)}$ is $10^3$.

\[
\text{RECIPROCAL } \frac{1}{10^2} = 10^{-2}
\]

\[
\text{THUS… } 10^5 = \frac{10^5}{10^2} = 10^5 \times 10^{-2}
\]

TO MULTIPLY

\[
= 10^{(5-2)} = 10^3
\]
Using the previous example we can simplify exponential expressions. The following is an example combining multiplication and division, which can be reduced to a simple addition problem.

\[
\frac{10^4 \times 10^7}{10^{-2} \times 10^3} = ?
\]

\[
\frac{10^4 \times 10^7}{10^{-2} \times 10^3} = 10^4 \times 10^7 \times 10^2 \times 10^{-3} = 10^{(4 + 7 + 2) - 3}
\]

\[
= 10^{10}
\]
Now, it is your turn to attempt some problems like these. Complete the problems below. The answers are on the next slide.

**Problem Set 3**

Complete the following expressions

1) $10^2 \times 10^3 = ?$
2) $10^2 \times 10^3 \times 10^6 = ?$
3) $10^5/10^4 = ?$
4) $10^2/10^{-1} = ?$
5) $10^5 \times 10^{-2} = ?$
6) $10^5 \times 10^6 \times 10^{-3} = ?$
7) $(10^8 \times 10^4)/10^{-2} = ?$
8) $(10^7 \times 10^3)/(10^{-2} \times 10^4) = ?$
9) $(10^{-5})/(10^{-2} \times 10^4 \times 10^{-6}) = ?$
10) $10^{-12}/10^{-10} = ?$
Solutions to Problem Set 3

1) $10^2 \times 10^3 = 10^5$
2) $10^2 \times 10^3 \times 10^6 = 10^{11}$
3) $10^5 / 10^4 = 10$ or $10^1$
4) $10^2 / 10^{-1} = 10^3$
5) $10^5 \times 10^{-2} = 10^3$
6) $10^5 \times 10^6 \times 10^{-3} = 10^8$
7) $(10^8 \times 10^4) / 10^{-2} = 10^{14}$
8) $(10^7 \times 10^3) / (10^{-2} \times 10^4) = 10^8$
9) $(10^{-5}) / (10^{-2} \times 10^4 \times 10^{-6}) = 10^{-1}$
10) $10^{-12} / 10^{-10} = 10^{-2}$
MULTIPLICATION OF COMPLEX EXPONENTIAL NUMBERS

EXAMPLE: \((1.2 \times 10^3) \times (1.2 \times 10^5)\)

STEP 1  
\[(1.2 \times 1.2) \times (10^3 \times 10^5)\]

STEP 2  
\[1.44 \times (10^3 + 5)\]

\[= 1.44 \times 10^8 \text{ round to } 1.4 \times 10^8\]

2 significant figures
EXAMPLE:

\[
\frac{(4.8 \times 10^4)}{(1.2 \times 10^2)}
\]

STEP 1

\[
\frac{4.8}{1.2} \times 10^4
\]

STEP 2

\[
4.0 \times 10^{4-2}
\]

= \(4.0 \times 10^2\)
SOLVING MORE COMPLICATED EXPRESSIONS

\[
\frac{(1.2 \times 10^4) \times (2.0 \times 10^{-2})}{(4.0 \times 10^{-3}) \times (6.0 \times 10^2)}
\]

SEPARATE

\[
\begin{align*}
1.2 & \times 2.0 \\
4.0 & \times 6.0
\end{align*}
\]

= 0.10

\[
\begin{align*}
1.2 & \times 2.0 \\
4.0 & \times 6.0
\end{align*}
\]

= 0.10

= 1.0

(Standard Exponential Form)
Now, it is your turn to attempt some problems like these. Complete the problems below. The answers are on the next slide.

**Problem Set 4**

Complete the following expressions. Express all your answers in standard exponential form.

1) \((1.2 \times 10^4) \times (2.0 \times 10^3) = ?\)
2) \((1.1 \times 10^{-4}) \times (3.0 \times 10^7) = ?\)
3) \((4.2 \times 10^5)/(3.0 \times 10^3) = ?\)
4) \((4.2 \times 10^{11})/(3.0 \times 10^{-4}) = ?\)
5) \([(1.44 \times 10^{-4}) \times (2.0 \times 10^6)]/(1.2 \times 10^{-3}) = ?\)
6) \([(4.6 \times 10^6) \times (2.2 \times 10^{-5})]/[(1.1 \times 10^4) \times (2.3 \times 10^{-7})] = ?\)
7) \((5.6 \times 10^{-5})/[(8.0 \times 10^{-2}) \times (7.0 \times 10^7)] = ?\)
Solutions to Problem Set 4

Complete the following expressions. Express all your answers in standard exponential form.

1) \((1.2 \times 10^4) \times (2.0 \times 10^3) = 2.4 \times 10^7\)

2) \((1.1 \times 10^{-4}) \times (3.0 \times 10^7) = 3.3 \times 10^3\)

3) \((4.2 \times 10^5)/(3.0 \times 10^3) = 1.4 \times 10^2\)

4) \((4.2 \times 10^{11})/(3.0 \times 10^{-4}) = 1.4 \times 10^{15}\)

5) \([(1.44 \times 10^{-4}) \times (2.0 \times 10^6)]/(1.2 \times 10^{-3}) = 2.4 \times 10^5\)

6) \([(4.6 \times 10^6) \times (2.2 \times 10^{-5})]/[(1.1 \times 10^4) \times (2.3 \times 10^{-7})] = 4.0 \times 10^4\)

7) \((5.6 \times 10^{-5})/[(8.0 \times 10^{-2}) \times (7.0 \times 10^7)] = 1.0 \times 10^{-11}\)
RULES: ADDITION AND SUBTRACTION

Rule One: Both exponentials must have the same exponent.

Rule Two: Transform the exponent of smaller number to that of the larger number.

Rule Three: Carry out addition or subtraction...multiply by the common exponent.
REVIEW

Multiplication

...Add Exponents

Division

...Subtract Exponent
REVIEW

MULTIPLICATION
Example:
\[(4.0 \times 10^2) \times (2.0 \times 10^6)\]

\[= (4.0 \times 2.0) \times (10^2 \times 10^6)\]

\[= 8.0 \times 10^8\]

DIVISION
Example:
\[\frac{4.0 \times 10^2}{2.0 \times 10^6}\]

\[= \frac{4.0}{2.0} \times \frac{10^2}{10^6}\]

\[= 2.0 \times 10^{2-6}\]

\[= 2.0 \times 10^{-4}\]